Physics IV
ISI B.Math
Backpaper Exam

Total Marks: 50
Time : 3 hours
Answer all questions

1.(**Marks: 4 + 2 + 2 = 10**)

(a) Two photons each have energy E. They collide at an angle θ and create a particle of mass M. What is M?

(b) Show that the sum of any two orthogonal spacelike vectors is spacelike.

(c) Show that a timelike vector and a null vector cannot be orthogonal.

2. (**Marks: 4 + 6 = 10**)

A one dimensional harmonic oscillator of mass m has potential energy $V(x) = \frac{1}{2}m\omega^2 x^2$. Consider the operators $a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - ip)$

It is given that $a^\dagger \psi_n = \sqrt{n+1}\psi_{n+1}$ and $a\psi_n = \sqrt{n}\psi_{n-1}$, where ψ_n is a solution of the time independent Schrödinger equation with energy E_n

(a) Given that a lowest energy ground state exists such that $a\psi_0 = 0$, find the normalized ground state wave function ψ_0 .

(b) Show that in the nth eigenstate of the harmonic oscillator, the average kinetic energy $< T >$ is equal to the average potential energy $< V >$ (the Virial theorem). What is the lowest value of the average kinetic energy that a quantum harmonic oscillator can have ? How does it contrast with the classical value ?

3. (**Marks: 2 + 4 + 4 = 10**)

A particle of mass m in the infinite square well $V(x) = 0$ for $0 \leq x \leq a$ and $V(x) = \infty$ otherwise has as its initial wave function an even mixture of the first two stationary states

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)]$$

(a) Normalize $\Psi(x, 0)$

(b) Find $\Psi(x, t)$ and $|\Psi(x, t)|^2$. Express the latter as a sinusoidal function of time. To simplify the result, let $\omega = \pi^2\hbar/2ma^2$.

(c) Compute $< x >$. Notice that it oscillates in time. What is the frequency of oscillation ? What is the amplitude of oscillation ?

4. (**Marks: 10**)

For a certain system, the operator corresponding to the physical quantity A does not commute with the Hamiltonian. It has eigenvalues α_1 and α_2 corresponding to properly normalized eigenfunctions

$$\phi_1 = \frac{(u_1 + u_2)}{\sqrt{2}}$$

$$\phi_2 = \frac{(u_1 - u_2)}{\sqrt{2}}$$

where u_1 and u_2 are properly normalized eigenfunctions of the Hamiltonian with eigenvalues E_1 and E_2. If the system is in the state $\psi = \phi_1$ at time $t = 0$, show that the expectation value of A at time t is

$$< A >= \left(\frac{\alpha_1 + \alpha_2}{2}\right) + \left(\frac{\alpha_1 - \alpha_2}{2}\right) \cos\left(\frac{|E_1 - E_2|t}{\hbar}\right)$$

5. (**Marks: 4 + 6 =10**)

(a) Show that in one dimension, the wavefunction $\psi(x)$ that is a solution to the time independent Schrödinger equation can always be taken to be real.

(b) Consider three observables $\hat{A}, \hat{B}, \hat{C}$ If it is known that $[\hat{B}, \hat{C}] = \hat{A}$ and $[\hat{A}, \hat{C}] = \hat{B}$, show that $\Delta(AB)\Delta(C) \geq \frac{1}{2}(A^2 + B^2)$ where $A^2, B^2 = < \hat{A}^2 >, < \hat{B}^2 >$ respectively and $\Delta(AB)$ and $\Delta(C)$ denote the uncertainties in $\hat{A}\hat{B}$ and $\hat{C}$ respectively.